Key

AP Calculus 1
More Problems

The following are six past tests. You can use these problems to help you prepare for the midterm exam. The best way to prepare for this exam is to complete as many problems as possible. Solving these problems as well as the problems chosen for you in the book "Cracking the AP Calculus Exam", and those found on the tests given during this semester provide will help you be well prepared for the midterm exam.

Name: ______________________________
September 3rd, 2010
AP Calculus 1, Mrs. Sulkes

Test #1 Form A, Q1
Functions and Graphing

NO CALCULATOR. ***Show your work in a neat and organized manner. Justify your answers for full credit.***

(+3 each) For #1 – 6, complete the blanks to make each statement true.

1. The domain of $f(x)=\ln(x^2-1)$ is $x<-1$ or $x>1$.

2. The range of $f(x)=e^{\frac{1}{x}}+2$ is $y>2$.

3. The coordinates of one hole on the graph of $f(x)=\dfrac{1}{\tan x}$ is $(0,0)$.

4. The vertical asymptote(s) of $f(x)=\dfrac{x+2}{x^2+5x+6}$ is/are $x=-3$.

$(x+2)(x+3)$

5. The x-intercept of $f(x) = \log_2(x) - 1$ is ___2___.

$\log_2 x = 1$

6. If (-2,4) is a point on the graph of $f(x)$, then ___$(\frac{7}{5}, 2)$___ is a point on the graph of $-\frac{1}{2}f(\frac{1-5x}{3}) + 4$.

$\frac{1}{3} - \frac{5}{3}x$

7. (+8) State whether the function is even, odd, or neither. Prove analytically. Then, in a sentence, describe the symmetry of $f(x)$.

$$f(x) = \frac{x(x^3 - \cos(2x))}{\sin x}$$

neither

b/c $f(-x) \neq f(x)$

and $f(-x) \neq -f(x)$

$$f(-x) = \frac{-x(-x^3 - \cos(-2x))}{\sin(-x)}$$

$$= \frac{-x(-x^3 - \cos(2x))}{-\sin x}$$

$$= \frac{-x(x^3 + \cos 2x)}{\sin x}$$

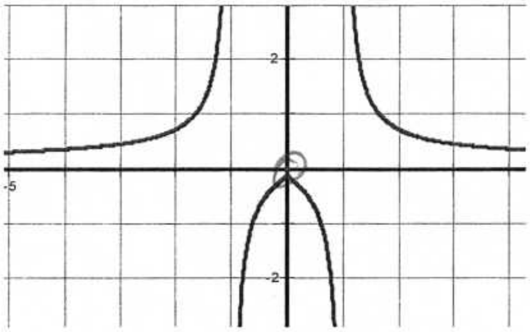
8. (+12) Write $f(x) = |1-x| + |2x-6|$ as a piecewise function. Show the analytical steps that lead to your answer.

$$|1-x| = \begin{cases} 1-x & x \le 1 \\ x-1 & x > 1 \end{cases}$$

$$|2x-6| = \begin{cases} 2x-6, & x \ge 3 \\ 6-2x, & x < 3 \end{cases}$$

$$f(x) = \begin{cases} 7-3x & x \le 1 \\ -x+5, & 1 < x < 3 \\ 3x-7, & x \ge 3 \end{cases}$$

9. (+2 each) Match each graph with one of the functions listed below.



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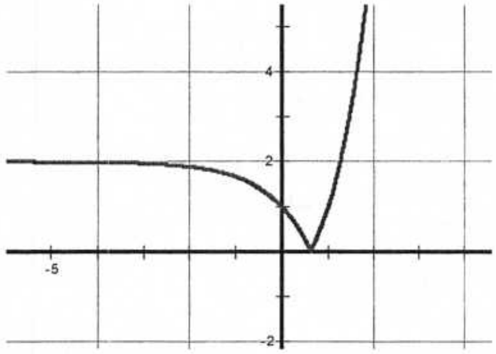

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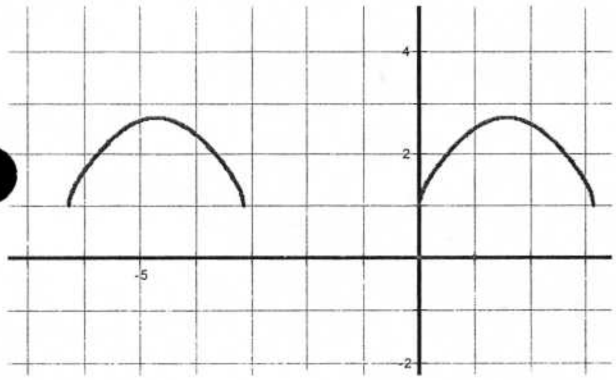

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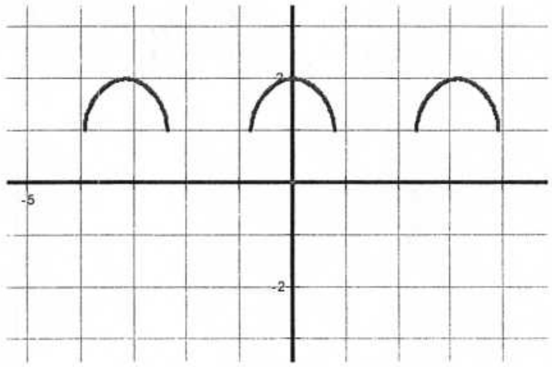


C

A. $f(x) = e^{\sqrt{\sin x}}$

B. $f(x) = \dfrac{1}{\ln(x^2)}$

C. $f(x) = \sqrt{\cos(2x)} + 1$

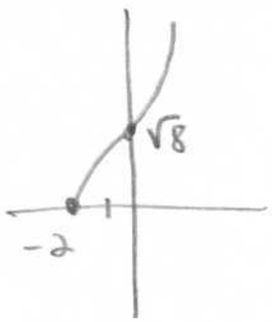
D. $f(x) = \left|3^x - 2\right|$

For each function in #10 – 11,

A. Sketch the graph.
B. Find any zeros and y-intercept. If none, so state.
C. State equations for any asymptotes. If none, so state.
D. Determine the coordinates of any holes. If none, so state.
E. State the domain and range.

10. (+12) $f(x)=\sqrt{x^3+8}$

A. Graph: You may show a preliminary graph as part of your work. Circle the final graph.



B. Zeros: -2 Y-intercept: $\sqrt{8}=2\sqrt{2}$

C. Asymptote(s): none

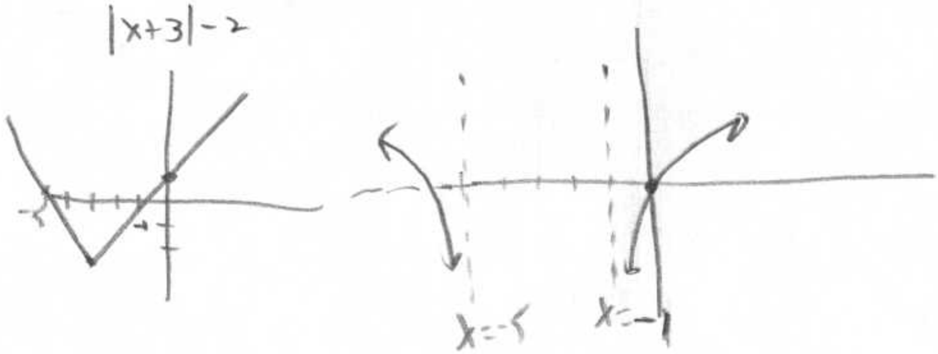
D. Hole(s): none

E. Domain: Range:

$x \geq -2$ $y \geq 0$

11. (+15) $f(x) = \ln(|x+3| - 2)$

A. Graph: You may show a preliminary graph as part of your work. Circle the final graph.



D. Zeros:

Y-intercept: $\ln 1 = 0$

$|x+3| - 2 = 1$

$|x+3| = 3$

$x+3 = 3$ or -3

$x = 0 \quad x = -6$

E. Asymptote(s):

$x = -5$

$x = -1$

D. Hole(s): none

E. Domain:

Range: $\mathbb{R}$

$x < -5$ or

$x > -1$

12. (+7) Given the rational function $f(x)=\dfrac{x^2-x}{x^3-4x^2+3x}$, $=\dfrac{x(x-1)}{x(x^2-4x+3)}=\dfrac{\cancel{x}\cancel{(x-1)}}{\cancel{x}(x-3)\cancel{(x-1)}}$

$=\dfrac{1}{x-3}$

Find:

A. Zeros:

none

B. Asymptote(s):

$x=3$

$y=0$

C. Hole(s):

$(0,-\frac{1}{3}$ and $(1,-\frac{1}{2})$

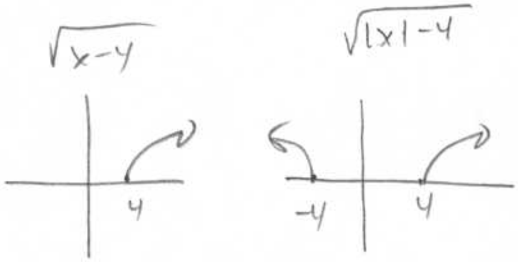
Pledge:

Name________________________________ AP Calc Q1

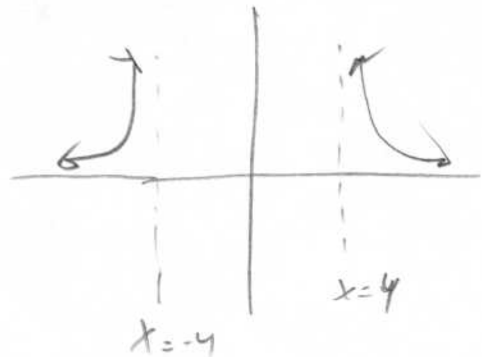
Test 2

Directions: Show all work in a neat and orderly manner. Partial credit may be awarded for partial solutions. You may not use a calculator on this test. Good luck.

1. Sketch the graph of. $f(x)=\dfrac{1}{\sqrt{|x|-4}}$. All graphs must include intercepts, asymptotes and key points.



$$\frac{1}{\sqrt{|x|-4}}$$



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2. a. Find the equation of the inverse of $f(x)=\dfrac{1}{\ln\left(\sqrt{x}-2\right)}$.

$$x=\frac{1}{\ln(\sqrt{y}-2)}$$

$$\ln(\sqrt{y}-2)=\frac{1}{x}$$

$$\sqrt{y}-2=e^{1/x}$$

$$\sqrt{y}=e^{1/x}+2$$

$$y=(e^{1/x}+2)^2$$

b. State the range of f^{-1}. = Domain of f(x) $\sqrt{x}-2>0$

$y>4$

$-\ |\ +$

4

3. Evaluate the following limits. If the limit does not exist write DNE. In this case, state if the limit tends to positive or negative infinity, if possible. NOTATION!

a. $\lim\limits_{x\to 5^-} \frac{x^2-4x-5}{|x-5|}$ $= \lim\limits_{x\to 5^-} \frac{\cancel{(x-5)}(x+1)}{-1\cancel{(x-5)}} = \frac{6}{-1} = -6$

b. $\lim\limits_{x\to 1} \frac{\frac{1}{x+2}-\frac{1}{3}}{x-1}$ $= \lim\limits_{x\to 1} \frac{3-x-2}{3(x+2)(x-1)} = \lim\limits_{x\to 1} \frac{\overset{-1}{\cancel{1-x}}}{3(x+2)\cancel{(x-1)}} = \frac{-1}{9}$

c. $\lim\limits_{x\to 0} x\csc 3x$ $= \lim\limits_{x\to 0} \frac{x}{\sin(3x)} = \frac{1}{3}$

d. $\lim\limits_{x\to -5^+} \frac{4x^2}{x+5}$ DNE

4. Suppose that $h(x)$ is a continuous function and the following table gives the values of $h(x)$ for the given values of x.

x	-4	-1	2	5
$h(x)$	2	-2	5	-1

Suppose k is a real number such that $-1 < k < 2$. According to the Intermediate Value Theorem, between what two numbers must c fall? between -1 and 2

Will there be more than one value of c that satisfies the theorem? Use a graph to explain your answer.

There will be at least 3.

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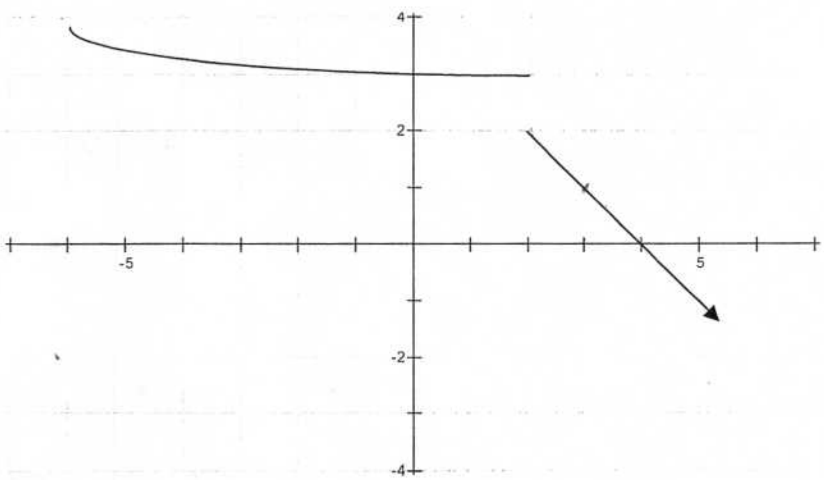
5. Use the graph below to answer the following questions. If the limit does not exist, write DNE and explain.



a. find $\lim_{x\to 2^+} f(x)$ = 2

b. find $\lim_{x\to 2^-} f(x)$ = 3

c. find $\lim_{x\to 2} f(x)$ DNE

d. Is f continuous at $x = 2$? Use the definition to continuity to explain.

no $\lim_{x\to 2} f(x)$ = DNE.

e. Is f continuous at $x = 3$? Use the definition to continuity to explain.

yes $f(3) = 1$

$\lim_{x\to 3} f(x) = 1$

Since $f(3) = \lim_{x\to 3} f(x)$

f(x) is continuous at $x = 3$

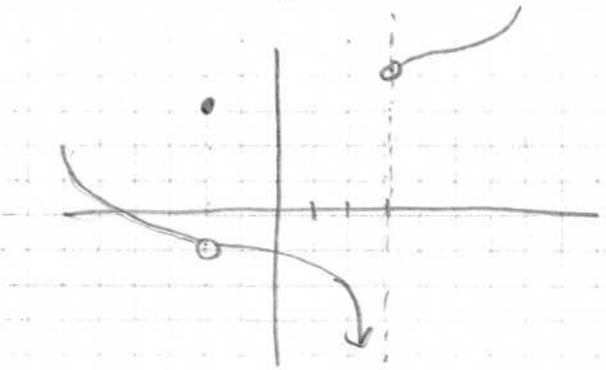
6. Sketch the function $g(x)$ such that the following conditions are met.

$\lim_{x \to 3^+} g(x) = 4$

$\lim_{x \to 3^-} g(x)$ DNE $(-\infty)$

$\lim_{x \to -2} g(x) = -1$

$g(-2) = 3$



Answers will vary.

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7. Find the values of a and b that would make f **continuous** for all values of x, given that $b > 0$.

$$f(x) = \begin{cases} \sqrt{a - 2x} & \text{if } x < 0 \\ 2b + x & \text{if } 0 \le x < 1 \\ ax^2 - b & \text{if } x \ge 1 \end{cases}$$

$\sqrt{a} = 2b$

$2b + 1 = a - b \rightarrow 3b + 1 = a$

$\sqrt{3b+1} = 2b$

$3b + 1 = 4b^2$

$0 = 4b^2 - 3b - 1$

$0 = (4b+1)(b-1)$

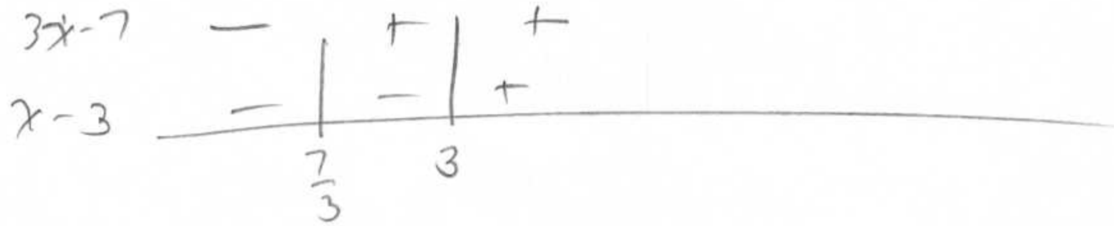
$\cancel{b = -1/4}$ $b = 1$

$\sqrt{a} = 2$

$a = 4$

8. State the domain of $g(x)=\sqrt{2+\dfrac{x-1}{x-3}}$.

$$\sqrt{\frac{2x-6+x-1}{x-3}} = \sqrt{\frac{3x-7}{x-3}}$$



$$\frac{3x-7}{x-3} \geq 0 \Rightarrow \quad x \leq 7/3 \text{ or } x > 3$$

Pledge: "On my honor, I have neither given nor received assistance on this test, nor do I know of another student who has given or received assistance on this test."

Signature:

3A	3A	3A	3A	3A

AP Calculus 1 **Name:**____

Test 3A, Quarter 1

Duty

Directions: No calculator is permitted on this test. Show all your work in a neat and orderly manner. Partial credit may be awarded for partial solutions.

1. **Evaluate the following limits. If the limit does not exist write "DNE" and explain.**

a. $\lim_{x\to 2}\sqrt{x-2} =$ DNE B/c $\lim_{x\to 2^-}\sqrt{x-2}$ DNE.

b. $\lim_{h\to 0}\dfrac{\sqrt{x+h}-\sqrt{x}}{h} =$ $\frac{1}{2}x^{-1/2}$

$f(x)=\sqrt{x}$

c. $\lim_{x\to -\infty}\dfrac{4}{e^x-e} =$ $= \dfrac{-4}{e}$

d. If $\lim_{x\to c} f(x) = -\frac{1}{2}$ and $\lim_{x\to c} g(x) = \frac{2}{3}$, then $\lim_{x\to c}[f(x) - g(x)] =$ $\frac{-1}{2} - \frac{2}{3}$

$\frac{-3-4}{6} = \frac{-7}{6}$

e. $\lim_{h\to 0} \frac{\sin^2 h}{1-\cos h} \cdot \frac{1+\cos h}{1+\cos h}$

$= \lim_{h\to 0} \frac{\sin^2 h(1+\cos h)}{1-\cos^2 h} = \lim_{h\to 0} \frac{\cancel{\sin^2 h}(1+\cos h)}{\cancel{\sin^2 h}} = 2$

2. Find the value(s) of x, if any, at which the function f is not continuous. State the type of discontinuity.

$f(x) = \frac{1}{|x^3 - 2x^2|}$ $x^2(x-2) = 0$ at $x = 0$ & $x = 2$

discontinuous at $x = 0$ and $x = 2$ → asymptotic.

3. Find a nonzero value for the constant k that makes $f(x) = \begin{cases} \dfrac{\sin(3x)}{x}, & x \neq 0 \\ 2+3k^2, & x = 0 \end{cases}$ continuous at x = 0.

$3 = 2 + 3k^2$

$1 = 3k^2$

$\frac{1}{3} = k^2$

$k = \pm\sqrt{1/3}$

4. Given $f(x) = \begin{cases} -x^4+3, & x \leq 2 \\ x^2+9, & x > 2 \end{cases}$. Is *f(x)* continuous at x = 2? Justify your answer.

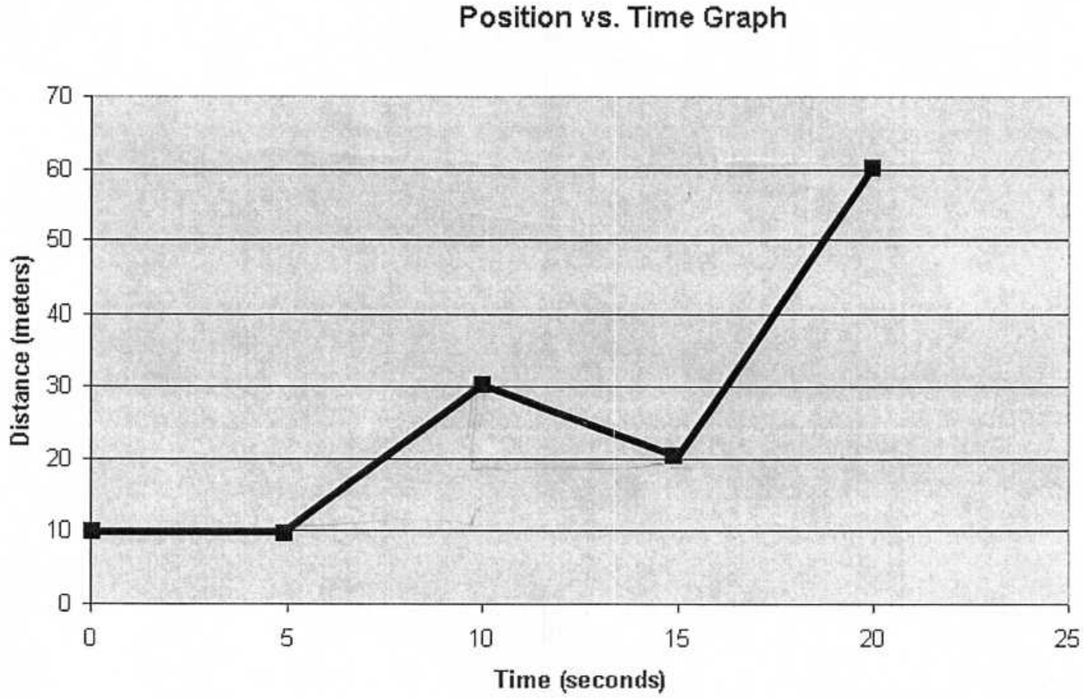
$f(2) = -16 + 3 = -13$

$\lim_{x \to 2^+} f(x) = 13 \qquad \lim_{x \to 2^-} f(x) = -13$

$\therefore \lim_{x \to 2} f(x)$ DNE

So f(x) is not continuous at x = 2

5. The graph below shows the position versus time curve of a particle. Use the information in the graph to answer the following questions.



a. What is the average velocity of the particle during the first twenty seconds? Be sure to include units.

$\frac{50}{20} = \frac{5}{2}$ m/sec

b. On what time interval is the particle moving the fastest? Justify your answer. Be sure to include units.

(0, 5) velocity = 0
(5, 10) velocity = 4
(10, 15) velocity = $^{-}2$

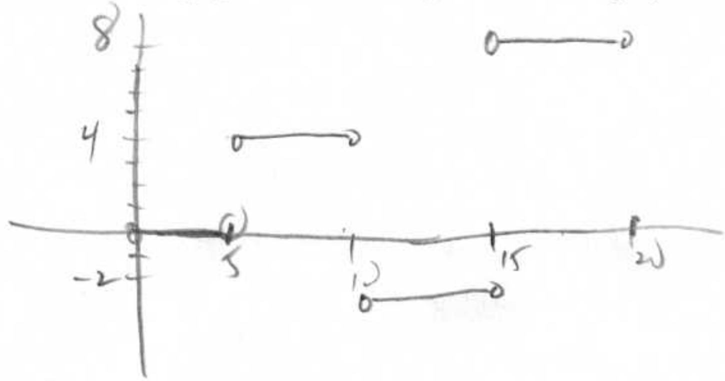
(15, 20) velocity = 8 m/sec

fastest on (15, 20)

c. What is the velocity of the particle at 12 seconds? Be sure to include units.

-2 m/sec

d. Sketch the graph of the velocity versus time graph for the particle.



6. Given $f(x) = \dfrac{1}{\sqrt{x-1}}$.

a. Using the limit definition of the derivative, find the derivative of *f(x)*.

$$\lim_{h\to 0}\left(\frac{1}{\sqrt{x+h-1}} - \frac{1}{\sqrt{x-1}}\right)\frac{1}{h} = \lim_{h\to 0}\frac{\sqrt{x-1}-\sqrt{x+h-1}}{\sqrt{x-1}\cdot\sqrt{x+h-1}}\cdot\frac{1}{h}$$

$$\lim_{h\to 0}\frac{\sqrt{x-1}-\sqrt{x+h-1}}{\sqrt{x-1}\cdot\sqrt{x+h-1}}\cdot\frac{\sqrt{x-1}+\sqrt{x+h-1}}{\sqrt{x-1}+\sqrt{x+h-1}}\cdot\frac{1}{h}$$

$$\lim_{h\to 0}\frac{x-1-(x+h-1)}{h(\sqrt{x-1}\cdot\sqrt{x+h-1})(\sqrt{x-1}+\sqrt{x+h-1})} = \lim_{h\to 0}\frac{-\cancel{h}}{\cancel{h}(\sqrt{x-1}\cdot\sqrt{x+h-1})((\sqrt{x-1})+\sqrt{x+h-1}}$$

$$= \frac{-1}{(x-1)(2\sqrt{x-1})} = \frac{1}{2(x-1)^{3/2}}$$

b. Find the slope of the tangent at x = 5.

$$f'(5) = \frac{1}{2(4)^{3/2}} = \frac{1}{16}$$

1A	1A	1A	1A	1A

AP Calculus 1 **Name:**____
Test 1 A, Quarter 2
Duty
74 points

Directions: No calculator is permitted on this test. Show all your work in a neat and orderly manner. Circle the correct choice. Partial credit may be awarded for partial solutions.

1. If $y = 2x^{\frac{5}{2}}\tan(x)$, then $\frac{dy}{dx} =$

$5x^{3/2}\tan(x) + 2x^{5/2}\cdot\sec^2 x$

$x^{3/2}(5\tan x + 2x\sec^2 x)$

a. $x^{\frac{3}{2}}\left(5\tan x + 2x\sec^2 x\right)$

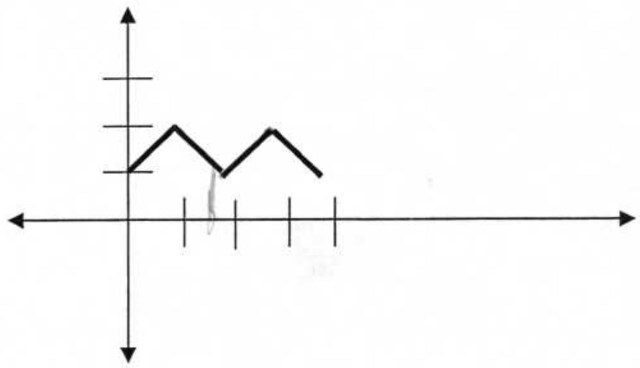
b. $5x^{\frac{3}{2}}\tan x - x\csc^2 x$

c. $5x^{\frac{5}{2}}\tan x + 2x^{\frac{3}{2}}\sec^2 x$

d. $x^{\frac{3}{2}}\left(5\tan x + x\sec^2 x\right)$

e. $x^2\tan x - 5x\csc^2 x$

2. The graph of $g(x)$ is sketched below on the interval [0,4].



Which of the following statements about $g(x)$ are (is) true?

I. There is some point c in the interval [0,4] such that $g'(c) = 0$. F

II. g is continuous at x = 2. T

III. $g'\left(\frac{3}{2}\right) < 0$. T

a. I only

b. II only

c. I and II only

d. II and III only

e. I, II, and III

3. $g(x) = |2x-1|$. Which of the following statements are (is) true?

I. $g(x)$ is continuous at $x = \frac{1}{2}$. T

II. $g(x)$ is differentiable at $x = \frac{1}{2}$. F

III. $\lim_{x \to \frac{1}{2}} g(x) = 0$. T

a. I only

b. I and II only

c. III only

d. I and III only

e. I, II, and III

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4. Find $\lim_{x \to 2} \frac{\ln x - \ln 2}{x-2} =$ $f(x) = \ln x$

$f'(x) = \frac{1}{x}$

$f'(2) = \frac{1}{2}$

a. $e^{\frac{1}{x}}$

b. e^2

c. $\frac{1}{2}$

d. e^x

e. $\frac{1}{x}$

5. If $y = x(\ln x)^2$, then $\frac{dy}{dx} =$ $(\ln x)^2 + 2(\ln x) \cdot \frac{1}{x} \cdot x$

a. $3(\ln x)$ $\ln x(\ln x + 2)$

b. $(\ln x)(2x + \ln x)$

c. $(\ln x)(2 + \ln x)$

d. $(\ln x)(2 + x\ln x)$

e. $(\ln x)(1 + \ln x)$

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6. The equation of the line tangent to the curve $y = \frac{kx+8}{k+x}$ at x = -2 is y = x + 4. What is the value of k?

a. -1

b. -3

c. 1

d. 3

e. 4

$$\frac{k(k+x) - 1(kx+8)}{(k+x)^2} = y'$$

$$\frac{k(k-2) - (-2k+8)}{(k-2)^2} = 1$$

$$\frac{k^2 - 2k + 2k - 8}{(k-2)^2} = 1$$

$$k^2 - 8 = k^2 - 4k + 4$$

$$-12 = -4k$$

$$\boxed{3 = k}$$

7. If $y = \cos^2 x - \sin^2 x$, then $y' =$ $-\sin(2x) \cdot 2$

↳ $\cos 2x$

a. -1

b. 0

c. $-2\sin(2x)$

d. $-2(\cos x + \sin x)$

e. $2(\cos x - \sin x)$

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8. If $f(x) = \dfrac{\sqrt{x+1}}{x^3+2}$, then $f'(0) =$

a. 0

$$\frac{\frac{1}{2}(x+1)^{-1/2}(x^3+2) - 3x^2(x+1)^{1/2}}{(x^3+2)^2}$$

b. $\dfrac{16}{9}$

c. $\dfrac{1}{4}$

$$\frac{\frac{1}{2}(2) - 0(1)^{1/2}}{((2)^2)} = \frac{1}{4} = \frac{1}{36}$$

d. $\dfrac{8}{9}$

e. None of the above. Write your solution in the blank.________________

9. If $f(x) = (g(x))^5, g(2) = -1, and\ f'(2) = 5$, find $g'(2)$.

$f'(x) = 5[g(x)]^4 \cdot g'(x)$

$5 = 5(-1)^4 \cdot g'(2)$

$1 = g'(2)$

a. -5

b. 0

c. $\frac{1}{5}$

d. 1

e. 5

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10. $f(x) = e^{\sin^2 x}$. $f'(x) =$ $e^{\sin^2 x} \; 2\sin x \cdot \cos x$

a. $e^{\sin^2 x}$

b. $2\sin x\, e^{\sin^2 x}$

c. $2\sin x \cos x\, e^{\sin^2 x}$

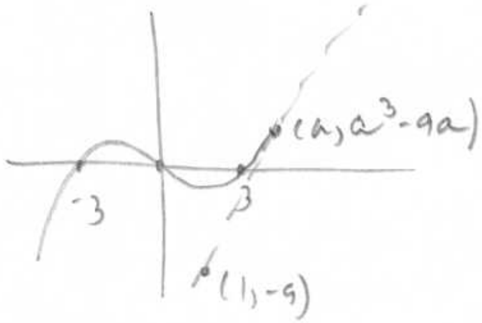
d. $e^{2\cos x}$

e. $e^{2\cos^2 x}$

Free Response Section:

$\frac{27}{8} - \frac{27}{2}$

1. Find the tangent lines to the curve $y = x^3 - 9x$ that pass through the point (1,-9). Note: (1, -9) is not on the curve. (12 points)



$y' = 3x^2 - 9$

$m = 3a - 9$

$m = \frac{a^3 - 9a + 9}{a - 1}$

$(3a^2 - 9) = \frac{a^3 - 9a + 9}{a - 1}$

$3a^3 - 3a^2 - 9a + 9 = a^3 - 9a + 9$

$2a^3 - 3a^2 = 0$

$a^2(2a - 3) = 0$

$a = 0 \quad a = 3/2$

pts $(0,0) \quad (3/2, -\frac{81}{8})$

Slope $= -9$ or $m = \frac{-9}{2}$

tangent lines

$y = -9x$ or

$y + \frac{81}{8} = \frac{-9}{2}(x - \frac{3}{2})$

2. Given $f(x) = \begin{cases} ax^3, & \text{if } x \le 2 \\ x^2 + b, & \text{if } x > 2 \end{cases}$, find a and b so that f(x) is continuous and differentiable. (6 points)

$f'(x) = \begin{cases} 3ax^2 \\ 2x \end{cases}$

$8a = 4 + b$

$12a = 4$

$a = \frac{1}{3}$

$\frac{8}{3} = 4 + b$

$-\frac{4}{3} = b$

Name: ______________________________

November 3rd, 2010

AP Calculus 1, Mrs. Sulkes

Test #2, Q2 Form A

Derivates

NO CALCULATOR. ***You must show the analytical work to justify each answer. Good luck!***

For #1 – 4, fill in the blanks to make each statement true:

1. The derivative of $y = \sec x$ is $\sec x \tan x$.

2. The function $f(x) = \dfrac{1}{x+2}$ is not differentiable at $x =$ -2.

3. The function $g(x) = \sqrt{3 - x^2}$ is continuous on the interval $[-\sqrt{3}, \sqrt{3})$.

4. The derivative of $y = \log x$ is $\dfrac{1}{x \ln 10}$.

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5. Show that the derivative of $y = \ln\left(\sqrt[3]{\frac{x-2}{x+2}}\right)$ is $\dfrac{4}{3(x^2-4)}$. Show all algebraic steps for full credit.

$$y = \frac{1}{3}\left(\ln(x-2) - \ln(x+2)\right)$$

$$y' = \frac{1}{3}\left[\frac{1}{x-2} - \frac{1}{x+2}\right]$$

$$y' = \frac{1}{3}\left(\frac{x+2-(x-2)}{(x-2)(x+2)}\right)$$

$$= \frac{1}{3}\left(\frac{4}{x^2-4}\right)$$

6. Show that the derivative of $y=\left(\sin^2\sqrt{x}\right)\left(\tan\sqrt{x}\right)$ is $\dfrac{\tan\sqrt{x}\left(\sin(2\sqrt{x})+\tan\sqrt{x}\right)}{2\sqrt{x}}$. Show all algebraic steps for full credit.

$y' = 2\sin\sqrt{x}\cdot\cos\sqrt{x}\cdot\frac{1}{2}x^{-1/2}\tan\sqrt{x} + \sec^2\sqrt{x}\cdot\frac{1}{2}x^{-1/2}\cdot\sin^2\sqrt{x}$

$y' = \frac{1}{2}x^{-1/2}\left[2\sin\sqrt{x}\cos\sqrt{x}\cdot\tan\sqrt{x} + \frac{1}{\cos^2\sqrt{x}}\sin^2\sqrt{x}\right]$

$= \frac{1}{2}x^{-1/2}\left[\sin 2\sqrt{x}\cdot\tan\sqrt{x} + \tan^2\sqrt{x}\right]$

$= \frac{1}{2}x^{-1/2}\tan\sqrt{x}\left[\sin 2\sqrt{x} + \tan\sqrt{x}\right]$

---

7. Find the derivative of $y=3^{x^2-2x+1}$.

$y' = 3^{x^2-2x+1}\cdot(2x-2)$

---

8. If $g(x)=\dfrac{e^{3x}}{\cos x}$, then $g'(1)=$

$g'(x) = \dfrac{3e^{3x}\cos x - -\sin x\cdot e^{3x}}{\cos^2 x}$

$g'(1) = \dfrac{3e^3\cos 1 + e^3\sin 1}{(\cos 1)^2}$

9. Find the value(s) of x, if any, at which the tangent line(s) to the graph of the function $f(x)=-3x+4e^{2x}$ is/are parallel to the line $y=-x+4$. Justify your answer with work.

$f'(x) = -3 + 8e^{2x}$

$-3 + 8e^{2x} = -1$

$8e^{2x} = 2$

$e^{2x} = \frac{1}{4}$

$2x = \ln(1/4)$

$x = \frac{\ln(\frac{1}{4})}{2}$

---

10. The $\lim\limits_{h\to 0} \dfrac{\cot^2(x+h)-\cot^2 x}{h}$ at $x=\frac{3\pi}{4}$ is:

$y = \cot^2 x$

$y' = 2\cot x \cdot -\csc^2 x$

$y'(\frac{3\pi}{4}) = 2\cot(\frac{3\pi}{4}) \cdot -[\csc(\frac{3\pi}{4})]^2$

$= 2(-1) \cdot -1(\sqrt{2})^2 = -2 \cdot -2 = 4$

---

11. What is the average rate of change on the interval (1,4) of the function $f(x)=\dfrac{1-x}{x+2}$?

$\frac{f(4)-f(1)}{3} = \frac{\frac{-3}{6} - 0}{3} = \frac{\frac{-1}{2}}{3} = \frac{-1}{6}$

12. Find the values of a and b so that f is differentiable (and continuous) for all values of x.

$$f(x)=\begin{cases} ax^2+b & \text{if } x<1 \\ 4a\ln x-2x & \text{if } x\geq 1 \end{cases}$$

$$f'(x)=\begin{cases} 2ax \\ \frac{4a}{x}-2 \end{cases}$$

$a+b=-2$

$2a=4a-2 \rightarrow -2a=-2$

$a=1$

$1+b=-2$

$b=-3$

---

13. If f and g are differential functions, $h(x)=f(g(x))+g(x)$, and $h'(x)=2(f'(g(x))+1)$,

then $g'(x)=$

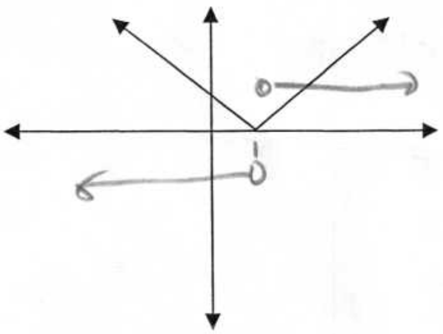
$h'(x)=f'(g(x))\cdot g'(x)+g'(x)=g'(x)[f'(g(x))+1]$

$2f'(g(x))+1=g'(x)[f'(g(x))+1)$

$$\boxed{\frac{2f'(g(x))+1}{f'(g(x))+1}=g'(x)}$$

---

14. Given the graph of $f(x)$ below, sketch the graph of $f'(x)$ on the same coordinate plane.



Pledge: ________________________________

Name: ______________________________

November 17th, 2010

AP Calculus 1, Mrs. Sulkes

Test #3, Q2 Form☺

NO CALCULATOR. ***For full credit, you must show the analytical work that leads to your answer.***

1. If $y=\sin(3^{2x})$, then $\frac{dy}{dx}=$ $\cos(3^{2x})\cdot 3^{2x}\ln 3\cdot 2$

2. If $f(x)=\sqrt{4\cos x+2}$, then $f'(\frac{\pi}{3})=$

$f'(x)=\frac{1}{2}(4\cos x+2)^{-1/2}(-4\sin x)$

$\boxed{f'(x)=-2\sin x(4\cos x+2)^{-1/2}}$

3. If $f(x)=x(\ln x)^2$, then $f'(e)=$

$f'(x)=(\ln x)^2+x\cdot 2\ln x\cdot\frac{1}{x}$

$=(\ln x)^2+2\ln x$

$f'(e)=1+2$

$=3$

4. If $y = \frac{1}{2}x\,\mathrm{arc\,cot}(e^{3x})$, then $\frac{dy}{dx} =$

$$y' = \frac{1}{2}\cot^{-1}(e^{3x}) + \frac{1}{2}x \cdot \frac{-1}{1+(e^{3x})^2} \cdot e^{3x} \cdot 3$$

$$= \frac{1}{2}\left[\cot^{-1}(e^{3x}) + \frac{\cancel{-}3 \cdot x e^{3x}}{1+e^{6x}}\right]$$

5. Use logarithmic differentiation to find $\frac{dy}{dx}$ of $y = (\sec x)^{\ln x}$ in terms of x. You do not need to simplify the derivative, just solve for $\frac{dy}{dx}$.

$$\ln y = \ln x \sec x$$

$$\frac{1}{y}y' = \frac{1}{x}\sec x + \sec x \tan x \cdot \ln x$$

$$y' = y\left[\frac{1}{x}\sec x + \sec x \tan x \cdot \ln x\right]$$

6. Let $f(t)=\frac{1}{t}$ for $t>0$. For what value of t is $f'(t)$ equal to the average rate of change of f on the closed interval [1,5]?

$$\frac{\frac{1}{5}-1}{5-1} = -1t^{-2}$$

$$\frac{\frac{-4}{5}}{4} = -1t^{-2}$$

$$-\frac{1}{5} = \frac{-1}{t^2}$$

$$t^2 = 5$$

$$t = \sqrt{5}$$

7. Use the given information to answer the questions below:

Let f and g be differentiable functions such that $f(1)=4$, $f(3)=-2$, $g(1)=3$, $f'(3)=-5$, $f'(1)=-4$, $g'(1)=-3$, and $g'(3)=2$.

a. If $h(x)=f(g(x))$, then $h'(1)=$

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$h'(1) = f'(g(1)) \cdot g'(1)$$

$$= f'(3) \cdot -3$$

$$= -5 \cdot -3 = 15$$

b. If $k(x)=\arctan(f(x))$, then $k'(3)=$

$$k'(x) = \frac{f'(x)}{1+[f(x)]^2}$$

$$k'(3) = \frac{f'(3)}{1+[f(3)]^2} = \frac{-5}{1+4} = -1$$

8. If $g(x)$ is the inverse of $f(x)$ and $f(x)=x^3+x+1$, find $g'(-1)$.

$x = y^3 + y + 1$

$1 = 3y^2y' + y'$

$-1 = y^3 + y + 1$

$0 = y^3 + y + 2$

$y = -1$

$1 = 3(-1)^2 y' + y'$

$1 = 4y'$

$\frac{1}{4} = y'$

9. For what value(s) of x does the graph of the function $x + y^2 = xy$ have a vertical tangent?

$1 + 2yy' = y + xy'$

$2yy' - xy' = y - 1$

$y'(2y - x) = y - 1$

$y' = \frac{y-1}{2y-x}$

Vertical tan at $2y = x$

$2y + y^2 = (2y)y$

$y^2 + 2y = 2y^2$

$0 = y^2 - 2y$

$0 = y(y-2)$

$y = 0$ $\quad y = 2$ $\quad x - y = 2x$

$x = 0$ $\quad x = 4$ $\quad y = x$

Vertical tangents at $x = 0$ and $x = 4$

10. Let $v(t) = 3t^2 - 12t$ be the velocity of the particle moving along the x-axis for time t, $t \geq 0$ in seconds.

a. In which direction does the particle begin moving and when does it turn around? Show the analytical work to support your answer.

$v(0) = 0 \qquad v(t) = 0 = 3t(t-4)$

particle moves left then changes directions at $t = 4$

$$\begin{array}{c|c|c} & - & + \\ \hline 0 & & 4 \end{array}$$

b. When is the particle speeding up between 0 and 5 seconds? Show the analytical work to support your answer.

$a(t) = 6t - 12$

$$\begin{array}{l|c|c|c|} a(t) & - & + & + \\ v(t) & - & - & + \\ \hline & 0 & 2 \quad 4 & 5 \end{array}$$

Speeding up $(0, 2)$ and $(4, 5)$

11. A particle moves along the x-axis in such a way that its position at time t is given by $x(t) = \dfrac{1-t}{1+t}$.

What is the velocity of the particle at time $t = 2$?

$$v(t) = \frac{-1(1+t) - 1(1-t)}{(1+t)^2}$$

$$= \frac{-1 - t - 1 + t}{(1+t)^2} = \frac{-2}{(1+t)^2}$$

$$v(2) = \frac{-2}{9}$$

12. Given f is continuous and differentiable at $x = 5$, find a and b.

$$f(x) = \begin{cases} x^2 + bx & x \le 5 \\ 5\sin(\frac{\pi}{2}x) & x > 5 \end{cases}$$

no a?

$25 + 5b = -5$

$5b = -30$

$b = -6$

BONUS: Given: $f(x) = e^{\csc x}$

Find $f'(x)$. Then find the zeros of $f'(x)$ on the closed interval $[2\pi, 4\pi]$:

$f'(x) = e^{\csc x} \cdot -\csc x \cot x$

$-e^{\csc x} \cdot \csc x \cot x = 0$

$\csc x = 0$ $\quad\quad$ $\cot x = 0$

$\varnothing$ $\quad\quad$ $\cos x = 0$

$x = \frac{5\pi}{2}, \frac{7\pi}{2}$

Pledge: ____________________

→ Def of Derivative: