A.P. Calculus 1 Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Rolle’s Theorem

**Rolle’s Theorem:**

Hypotheses:

1. *f(x)* is continuous on the closed interval [a,b]
2. *f(x)* is differentiable on the open interval (a,b)
3. *f(a) = f(b)*

Conclusion: There exists a *c* on the (a,b) such that 

**Exploration of Rolle’s Theorem:**

**Part 1:**

Plot the points (0,5) and (3,5).

Construct a segment through these two points.

1. a. Is the segment drawn a continuous function on the interval (0,3)? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

b. Is the segment drawn a differentiable function on the interval (0,3)? \_\_\_\_\_\_\_\_\_\_\_\_\_\_

c. Is f(0) = f(3)? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

If yes to all of the above, at what x-coordinates on the interval (0,3) is the derivative of the

function equal to 0?

**Part 2:**

Now, construct a parabola through the points. Write an equation for your parabola. Hint:

Choose a vertex, then write the equation. Answers will vary.

1. Is your function continuous on the interval (0,3)? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Is your function differentiable on the interval (0,3)? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. Is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

If yes to all of the above, at what x-coordinates on the interval (0,3) is the derivative of the

function equal to 0?

**Exploration of the Hypotheses of Rolle’s Theorem**

**Rolle’s Theorem :**

If

1. *f(x)* is continuous on the closed interval [a,b]
2. *f(x)* is differentiable on the open interval (a,b) and
3. *f(a) = f(b)*

then, there exists a value *c* on the interval (a,b) such that 

**Part 1: Is it necessary that the function be continuous?**

Suppose that the function is not continuous. Let , where .

1. Is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. Is there a value *c* on the interval (1,3) , such that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. Can Rolle’s Theorem be applied? Explain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Does this mean that Rolle’s Theorem fails? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Part 2: Is it necessary for the function to be differentiable?**

Suppose the function is continuous but not differentiable everywhere in the open interval. Let , where .

1. Is continuous on (1, 3)? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. At what value of *x* is the function not differentiable? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. Is \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. Is there a value *c* on the interval (1,3) , such that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
5. Can Rolle’s Theorem be applied? Explain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Does this mean that Rolle’s Theorem fails? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Part 3: Is it necessary that , where *a* and *b* are the endpoints of the interval?**

Suppose .

1. Is *f(x)* continuous on the interval (1,3)? \_\_\_\_\_\_\_\_\_
2. Is *f(x)* differentiable on the interval (1,3)? \_\_\_\_\_\_\_\_\_
3. Is  Show the values. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. Is there a value *c* on the interval (1,3) , such that \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
5. If any of the 3 hypotheses of Rolle’s Theorem is violated, can you guarantee that there is a value *c* which satisfies the equation \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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Mean Value Theorem

**The Mean Value Theorem**

Hypotheses:

1. *f(x)* is continuous on the closed interval [a,b]
2. *f(x)* is differentiable on the open interval (a,b)

Conclusion: There exists a value *c* on the interval (a,b) such that 

**Exploration of the Mean Value Theorem**

**Part 1:**

1. Graph the function .
2. Is continuous on the interval (0,3)? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. Is differentiable on the interval (0,3)? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. Choose two points on the graph of , one at *x = 0* and the other at *x = 3* and construct a secant line through the points.
5. Draw a line tangent to *f(x)* that is parallel to the secant line. Calculate the coordinates where the tangent line intersects the function. Provide your work and a sketch of the graph of the functions and the secant and tangent line in the space below. Write your equations for the tangent and secant lines.
6. Is there a value *c* on the interval (0,3) such that 

If yes, find *c*.

**Exploration of the Hypotheses of Mean Value Theorem**

**Mean Value Theorem**

If

1. *f(x)* is continuous on the closed interval [a,b]
2. *f(x)* is differentiable on the open interval (a,b)

then, there exists a value *c* on the interval (a,b) such that 

**Part 1: Is it necessary that the function be continuous on the interval?**

1. Let , where .
2. Is *f(x)* continuous on (0,3)? Explain. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. Construct a secant line passing through the points at x = 0 and at x = 3 and calculate the slope of the secant, that is 
4. Is there a value *c* on the interval (0,3) such that 

If yes, find *c*. If no, why not?

1. Can the Mean Value Theorem be applied? Explain \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Does this mean that the Mean Value Theorem fails? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Part 2: Is it necessary that the function be differentiable on the open interval?**

1. Let , where .
2. Graph the function.
3. Is continuous on the interval (0,9)? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
4. Is *f(x)* not differentiable at every value on (0,9)? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
5. Construct a secant line passing through the points at x = 0 and at x = 9 and calculate the slope of the secant, that is 
6. Is there a value *c* on the interval (0,3) such that 

If yes, find *c*. If no, why not?

1. If the continuity or differentiability hypothesis is violated, can you guarantee that there is a value *c* on the interval (a,b) which satisfies the equation  \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Practice:**

1. On the interval [-5,5], there is no point at which the derivative of *f* (*x*) =|*x*| is equal to zero, even though

*f* (- 5) = *f* (5) . Is this a contradiction of Rolle's Theorem? Explain your answer.

1. Find the number *c* that satisfies Rolle's Theorem for *f* (*x*) = *sin*(*x*) on the interval 
2. An SUV enters the Florida Turnpike at noon and heads north. A camera takes a picture of the SUV at 1:50, 121 miles from when the SUV first entered the turnpike. Prove that at some moment the SUV was going over 65 mph.
3. Suppose that and  for all values of *x*. How large can *f(2)* possibly be?