**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**A.P. Calculus, Mrs. Sulkes**

**February, 4th, 2013**

**Area**

The whole second semester of calculus is based on the concept of area. This exercise will help you to explore methods to find the area between a curve and the x – axis.

**Problem 1**

Let , where ..



1. Calculate the area of the triangle formed by the function and the restricted domain. This is the exact area.
2. Of course, not all functions produce a definite geometric figure. Let’s explore a method for approximating the area. Divide the area under the curve using 4 rectangles which hit the function on the upper right corner of the rectangle (see figure). This method will be called a Right-Hand –Rectangle approximation.



Assume that the four rectangles pictured above have equal width. Calculate the area of each rectangle, and then add the areas. Is this area an over or under approximation of the actual area of the triangle?

1. Of course the more rectangles you break the area into, the closer to the actual area you will be. Redo the problem above, using 10 rectangles.

Assume that the rectangles have equal width. What is the width of each rectangle?

What are the heights of the rectangles?

Add the areas of the rectangles. Is this total and over or under approximation?

1. Of course, the best number of rectangles to use is an infinite number. Let’s explore sigma notation.

If you divide the area into number of rectangles of equal width,

1. What is the width of each rectangle?
2. What are the x-coordinates you will use to calculate the height of each rectangle? Assume that the upper right corner of each rectangle lies on the function.











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1. What are the heights of each rectangle?







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1. Set up the sigma notation for the sum of the areas of the rectangles. Then, take the limit as .

**Problem 2**

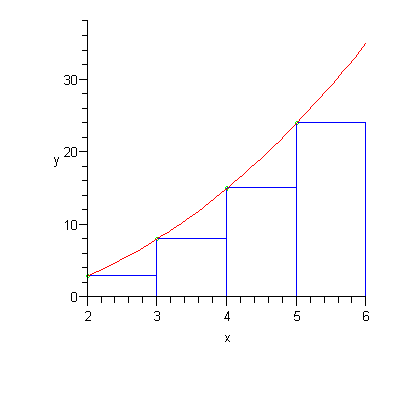
Let .

Suppose I want to find the area between the x-axis and the curve from x = 2 to .

Today you will explore three different ways to do this.

1. **Left Endpoint**

You can approximate the area beneath the curve by finding the area of n rectangles. Suppose you wish to approximate the area using 4 rectangles of equal width. Find the sum of the areas of each rectangle.



Now divide the area into 10 rectangles, and find the approximate area.

What is the width of each rectangle?

What are the heights of the rectangles?

Add the areas of the rectangles. Is this total an over or under approximation?

1. **Right Endpoint**

Now use the **right endpoint** of each interval to calculate the height of the rectangles. Find the sum of the areas of each rectangle. Approximate the area using 10 rectangles.

Sketch the graph and a few of the rectangles.

What is the width of each rectangle?

What are the heights of the rectangles?

Add the areas of the rectangles. Is this total an over or under approximation?

III. **Midpoint**

Finally, use the **midpoint** of each interval to find the height of the rectangles. This means that the rectangle hits the function at the midpoint of its top horizontal side. Approximate the area using 10 rectangles.

Sketch the graph and a few of the rectangles.

What is the width of each rectangle?

What are the heights of the rectangles?

Add the areas of the rectangles. Is this total an over or under approximation?

**V.** Of course, the best number of rectangles to use is an infinite number. Assume that the rectangles hit the function at its upper right corner (Right-Hand- Rectangle).

If you divide the area into number of rectangles of equal width,

1. What is the width of each rectangle?
2. What are the x-coordinates you will use to calculate the height of each rectangle? Assume that the upper right corner of each rectangle lies on the function.











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1. What are the heights of each rectangle?







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1. Set up the sigma notation for the sum of the areas of the rectangles. Then, take the limit as .