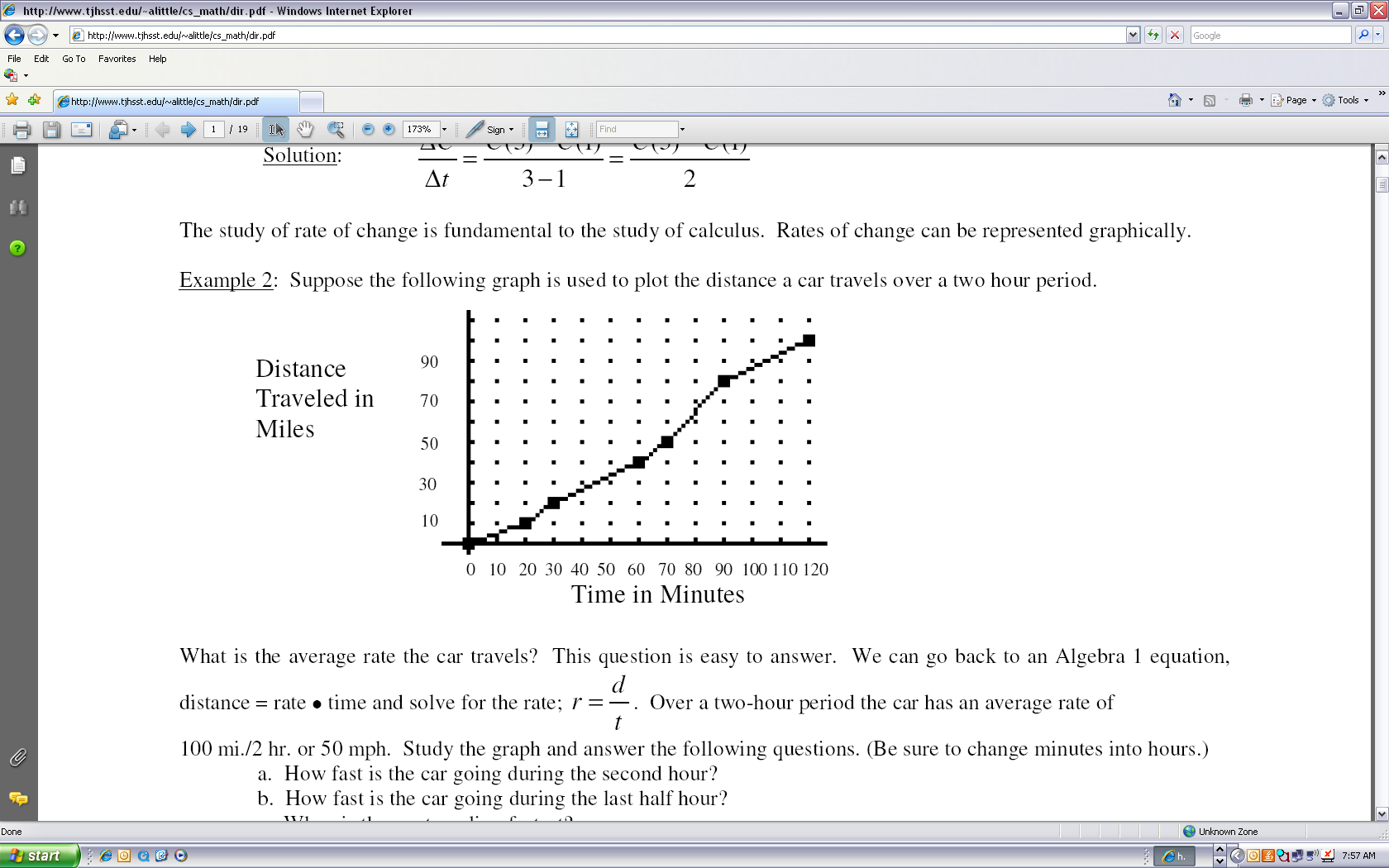
A.P. Calculus

Average Rate of Change

Example 1: Suppose the following graph is used to plot the distance a car travels over a two hour period.

What is the average rate the car travels?

Over a two-hour period the car has an average rate of 100 mi./2 hr. or 50 mph. Study the graph and answer the following questions. (Be sure to change minutes into hours.)



a. How fast is the car going during the second hour?

b. How fast is the car going during the last half hour?

c. When is the car traveling fastest?

d. When is the car traveling slowest?

e. What is the slope of the graph between ***t*** = 90 and ***t*** = 120?

f. What is the speed of the car at ***t*** = 50 minutes?

g. What is the speed of the car at ***t*** = 72 minutes?

Example 2: Suppose the position function for a free falling object is given by the equation, where t is time measured in seconds and s(t) is the height the object is off the ground measured in meters. Find the average velocity of the object between 0 and 2 seconds.

Example 3: Using the position function given in example 2 above, find the average velocity for each of the following time intervals.

a. From 1 to 2 seconds.

**b. What is the velocity at 1 second?**

Example 4:

|  |  |
| --- | --- |
| h = number of hours | T(h)= Temperature oC |
| 0 | 6.5 |
| 1 | 6.1 |
| 2 | 5.6 |
| 3 | 4.9 |
| 4 | 4.2 |
| 5 | 4.0 |
| 6 | 4.0 |
| 7 | 4.8 |
| 8 | 6.1 |
| 9 | 8.3 |

The T(h) (degrees Celsius) in the table are the temperature readings recorded every hour starting at midnight.

1. Find the average rate of change of temperature from 5am to 8 am.
2. Find the average rate of change of temperature from 7am to 8 am.
3. Estimate the ***instantaneous rate of change*** of temperature at 8am.

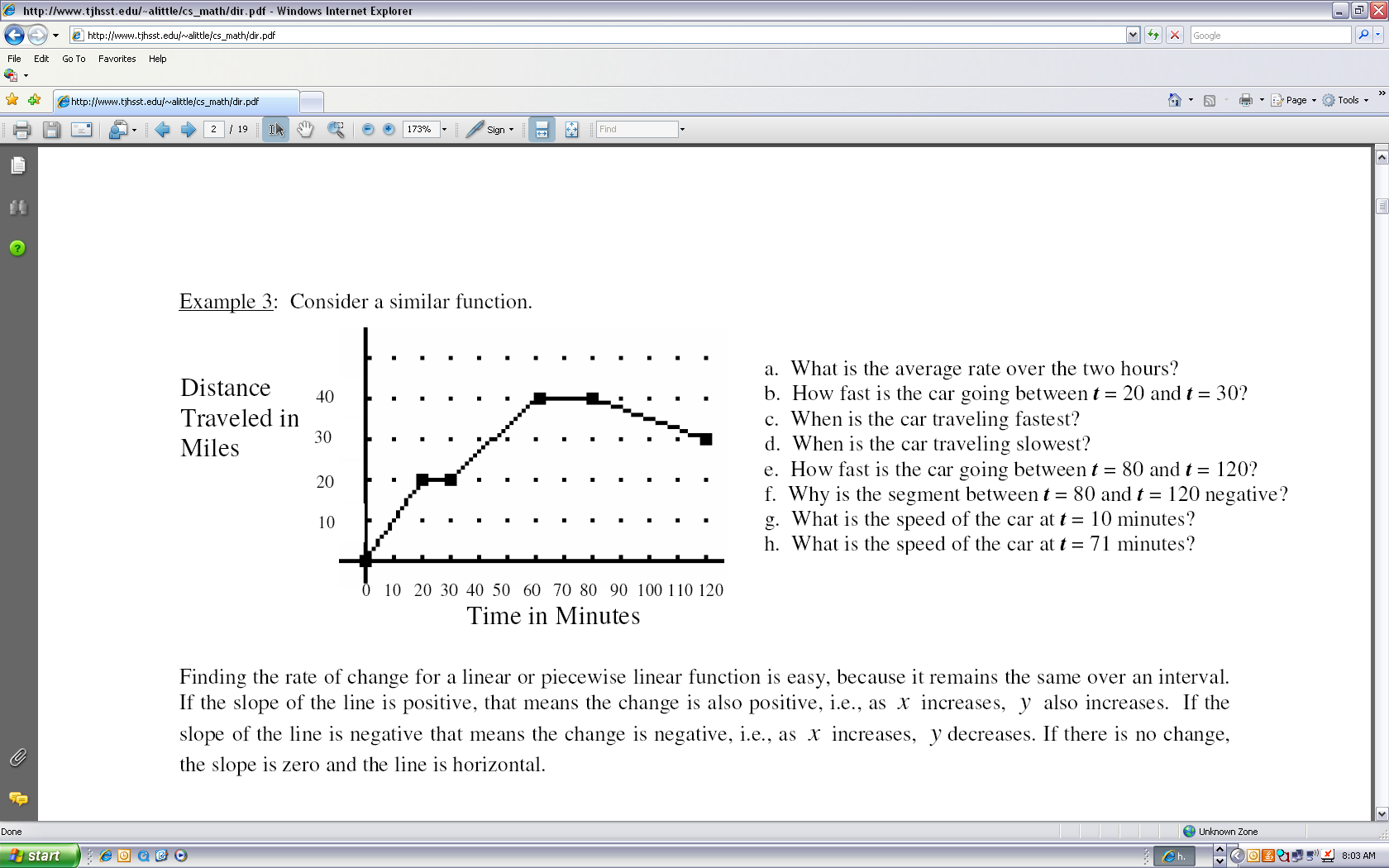
Example 5:

Given 

a. Using the limit definition of derivative, find 

b. Find the slope of the tangent at x = -2.

Example 2: Consider the function.



a. What is the average rate over the two hours?

b. How fast is the car going between ***t*** = 20 and ***t*** = 30?

c. When is the car traveling fastest?

d. When is the car traveling slowest?

e. Why is the velocity of the car between ***t*** = 80 and ***t*** = 120 negative?

f. How fast is the car going between ***t*** = 80 and ***t*** = 120?

g. What is the speed of the car at ***t*** = 10 minutes?

h. What is the speed of the car at ***t*** = 71 minutes?

Example 3:

Draw a **distance-time graph** representing the motion described on the grids provided. Be sure to label axes scale appropriately. A distance-time graph is often called a **position-time graph**.

1. Drive 60 mph for 10 minutes; stop for 10 minutes; drive 30 mph for 20 minutes. (Don't forget to change minutes to hours.)
2. Drive 30 mph for 20 minutes; turn around and drive 30 mph back towards your starting point for 10 minutes; turn around and walk away from your starting point again for 10 minutes.
3. Mrs. Duty went skiing. She waited for the ski lift at the bottom of the hill for 10 minutes, rode up on the ski lift at a constant speed of 2 mph for 3 minutes and then skied downhill for the next 12 minutes, going faster and faster until she fell.

For the same scenarios in a - c above, draw a **velocity-time graph** representing the motion described on the grids provided. Draw the horizontal axis. Be sure to label axes appropriately with units and scale.

Example 6: Suppose an object is thrown up from the top of a 96 foot platform and that its path is defined by the equation *s(t)* 

a. What is the average speed from ***t*** = 0 to ***t*** = 2.5 seconds?

b. What is the speed at ***t*** = 2.5 seconds?

c. What is the average speed from ***t*** = 2.5 to ***t*** = 6 seconds?

For each of the following, calculate the rate of change for the given situation. Be sure to include units.

1. The area (***A***) of a square is decreasing as the length of the side (***s***) decreases from ***s*** = 4 in. to ***s*** = 1 in.

2. The volume (***V***) of a sphere decreases as the radius (**r**) of the sphere decreases from ***s*** = 9 cm. to ***s*** = 5

cm.

3. The surface area (***A***) of a cube increases as the length of the side (***s***) of the cube decreases from 3 ft. to 8

ft.

4. The area of an equilateral triangle (***A***) increases as the length of a side (***s***) decreases from ***s*** = 3 cm. to ***s*** =

12 cm.