**Name:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**AP Calculus 1, Mrs. Sulkes**

**October 10th, 2011**

**3.2 Rates of Change – an Introduction**

**Average Rate of Change: (Slope of the Secant Line)**

|  |
| --- |
|  |

**Slope =**

**Instantaneous Rate of Change: (Slope of Tangent Line or Derivative)**

**Slope =**



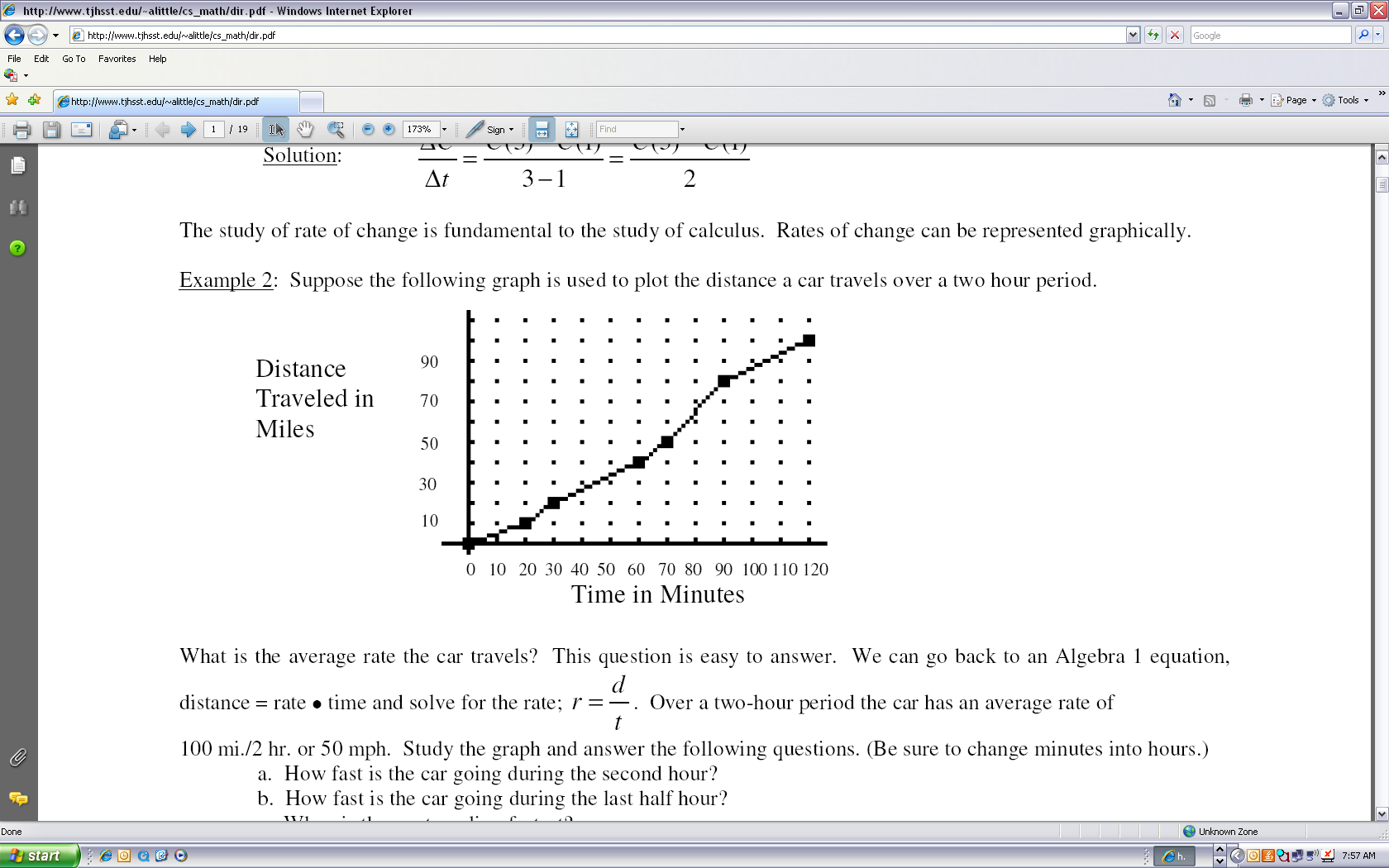
**Example 1:**

First, find the average rate of change of the function  on [0,2].

Next, find the instantaneous rates of change at the endpoints of the interval.

**Example 2:** Suppose the following graph is used to plot the distance a car travels over a two hour period. What is the average rate the car travels?

Over a two-hour period the car has an average rate of 100 mi./2 hr. or 50 mph. Study the graph and answer the following questions. (Be sure to change minutes into hours.)



a. How fast is the car going during the second hour?

b. How fast is the car going during the last half hour?

c. When is the car traveling fastest?

d. When is the car traveling slowest?

e. What is the slope of the graph between ***t*** = 90 and ***t*** = 120?

f. What is the speed of the car at ***t*** = 50 minutes?

g. What is the speed of the car at ***t*** = 72 minutes?

**Example 3:** The position function for a free falling object is given by the equation,  where t is time and s(t) is the height the object is off the ground. (g is acceleration due to gravity and is -9.8 meters per second per second or -32 feet per second per second.  is initial velocity and  is initial height. ) If the initial velocity of a free falling object is 0 and the initial height is 50 meters,

1. find the average velocity of the object between 0 and 2 seconds.
2. find the average velocity from 1 to 2 seconds.

c. find the velocity at 1 second?

d. find the velocity of the object at impact on the ground?

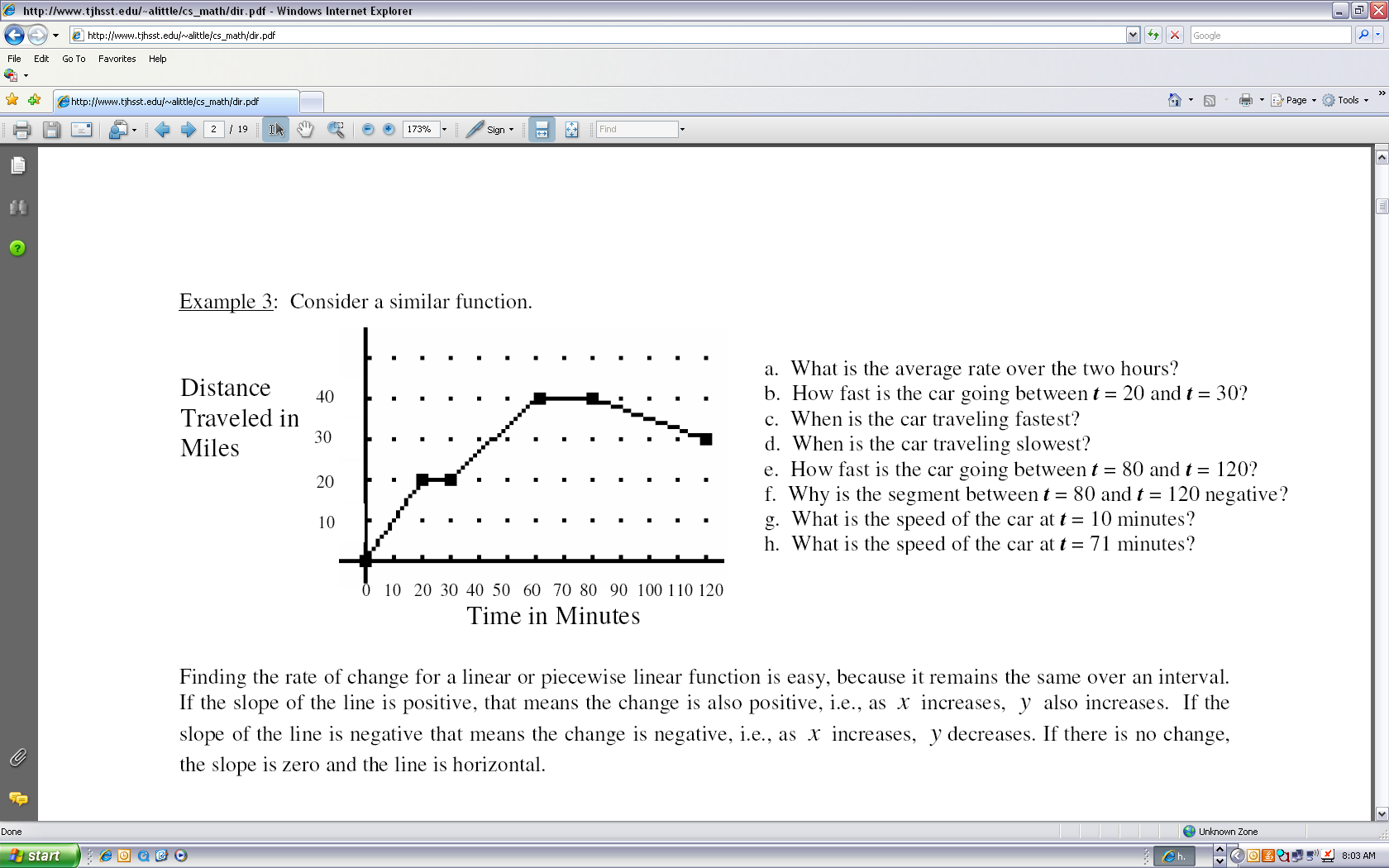
**Example 4:**

|  |  |
| --- | --- |
| h = number of hours | T(h)= Temperature oC |
| 0 | 6.5 |
| 1 | 6.1 |
| 2 | 5.6 |
| 3 | 4.9 |
| 4 | 4.2 |
| 5 | 4.0 |
| 6 | 4.0 |
| 7 | 4.8 |
| 8 | 6.1 |
| 9 | 8.3 |

The T(h) (degrees Celsius) in the table are the temperature readings recorded every hour starting at midnight.

1. Find the average rate of change of temperature from 5am to 8 am.
2. Find the average rate of change of temperature from 7am to 8 am.
3. Estimate the ***instantaneous rate of change*** of temperature at 8am.

**Example 5:** Consider the function.



a. What is the average rate over the two hours?

b. How fast is the car going between ***t*** = 20 and ***t*** = 30?

c. When is the car traveling fastest?

d. When is the car traveling slowest?

e. Why is the velocity of the car between ***t*** = 80 and ***t*** = 120 negative?

f. How fast is the car going between ***t*** = 80 and ***t*** = 120?

g. What is the speed of the car at ***t*** = 10 minutes?

h. What is the speed of the car at ***t*** = 71 minutes?

**Example 6:**

Draw a **distance-time graph (hours versus miles)** representing the motion described. Be sure to label axes scale appropriately. A distance-time graph is often called a **position-time graph**.

1. Drive 60 mph for 10 minutes; stop for 10 minutes; drive 30 mph for 20 minutes. (Don't forget to change minutes to hours.)
2. Drive 30 mph for 20 minutes; turn around and drive 30 mph back towards your starting point for 10 minutes; turn around and walk away from your starting point again for 10 minutes.
3. Mrs. Duty went skiing. She waited for the ski lift at the bottom of the hill for 10 minutes, rode up on the ski lift at a constant speed of 2 mph for 3 minutes and then skied downhill for the next 12 minutes, going faster and faster until she fell.

For the same scenarios in a - c above, draw a **velocity-time graph** representing the motion described. Draw the horizontal axis. Be sure to label axes appropriately with units and scale.

**Example 7:** Suppose an object is thrown up from the top of a 96 foot platform and that its path is defined by the equation *s(t)* =

a. What is the average speed from ***t*** = 0 to ***t*** = 2.5 seconds?

b. What is the speed at ***t*** = 2.5 seconds?

c. What is the average speed from ***t*** = 2.5 to ***t*** = 6 seconds?

**Example 8.**

For each of the following, calculate the rate of change for the given situation. Be sure to include units.

a. The area (***A***) of a square is decreasing as the length of the side (***s***) decreases from ***s*** = 4 in. to ***s*** = 1 in.

b. The volume (***V***) of a sphere decreases as the radius (**r**) of the sphere decreases from ***s*** = 9 cm. to ***s*** = 5

cm.

c. The surface area (***A***) of a cube increases as the length of the side (***s***) of the cube decreases from 3 ft. to 8

ft.

d. The area of an equilateral triangle (***A***) increases as the length of a side (***s***) decreases from ***s*** = 3 cm. to ***s*** =

12 cm.